

# Cosmological consequences of the brane/bulk interaction

C. van de Bruck<sup>1\*</sup>, M. Dorca<sup>2†</sup>, C.J.A.P. Martins<sup>1,3‡</sup>, M. Parry<sup>4§</sup>

<sup>1</sup> *Department of Applied Mathematics and Theoretical Physics, Center for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, U.K.*

<sup>2</sup> *Department of Physics, Box 1843, Brown University, Providence, RI 02912, USA*

<sup>3</sup> *Centro de Astrofísica, Universidade do Porto, Rua das Estrelas s/n, 4150-762 Porto, Portugal*

<sup>4</sup> *Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BZ, U.K.*

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We investigate cosmological consequences arising from the interaction between a homogeneous and isotropic brane-universe and the bulk. A Friedmann equation is derived which incorporates both the brane and bulk matter contributions, which are both assumed to be of arbitrary fluid form. In particular, new terms arise which describe the energy flow onto (or away from) the brane, as well as changes of the equation of state in the bulk. We discuss Randall–Sundrum type models as well as dilatonic domain walls and carefully consider the conditions for stabilizing the induced gravitational constant. Furthermore, consequences for cosmological perturbations are analyzed. We show that, in general, super-horizon amplitudes are not constant.

## I. INTRODUCTION

The idea that the universe is a brane embedded in a higher dimensional space has attracted a lot of attention in the last two years [1]—for earlier proposals see [2]. It was found that the Friedmann equation on the brane contains corrections to the usual four-dimensional equation. In particular a term  $H \propto \rho$  was found [3]. This term is observationally problematic (*e.g.* for nucleosynthesis, among other things), but the model is consistent if one considers a cosmological constant in the bulk and the tension on the brane, which leads to a cosmological version of the Randall–Sundrum scenario [4] of warped geometries [5].

On the other hand, particle physics models predict the existence of scalar fields, both on the brane and in the bulk. In fact, in heterotic M-theory, the 11-dimensional strong coupling limit of the  $E_8 \times E_8$  heterotic string theory, one particular bulk scalar field measures the deformation of the Calabi–Yau space and can not arbitrarily set to be zero. It is actually one of the aims of string cosmology to investigate the consequences of the evolution of these moduli fields. Two interesting examples are: i) in brane world scenarios, scalar fields are thought to stabilize the fifth dimension (see *e.g.* [6]); ii) the energy of bulk scalar field(s) might flow onto or away from the brane universe and thereby modify the expansion dynamics.

But cosmology is more than the evolution of the background. One of the biggest puzzles in cosmology is the origin and evolution of the structures we see in our universe. In the last twenty years it became a working hypothesis to search for the origin of these structures in the physics of the micro-cosmos. The most popular paradigm is the inflationary universe [7]. Here one speculates that the seeds of the structures are quantum fluctuations in the inflaton field which are stretched onto macroscopic length scales during a ‘super-luminal’ expansion epoch (commonly called inflation), where they become classical. Inflation leads to definitive observational signals, *e.g.* in the form of peaks and valleys in the spectrum of microwave background anisotropies and there is very good hope to check and severely constrain the parameter space of inflationary models with ongoing balloon experiments and future satellite experiments. In the context of brane world scenarios it is crucial to investigate if new effects appear and how they influence the evolution of perturbations. This may then provide a way to test these theories.

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\*Electronic address: C.VanDeBruck@damtp.cam.ac.uk

†Electronic address: dorca@het.brown.edu

‡Electronic address: C.J.A.P.Martins@damtp.cam.ac.uk

§Electronic address: m.parry@ic.ac.uk

At the time of writing, brane world cosmology is still in its infancy. Issues of inflationary model building in this context have just begun to be considered, and so far the evolution of density fluctuations has been relatively neglected in these scenarios. Recently, investigations started in this direction, see [8] and [9]. Such a formalism allows for an investigation of the full 5D field equation for perturbations of first order.

In this paper we turn our attention to the interaction between the bulk and the brane, which is another non-trivial aspect of brane world theories. In particular, we will discuss the flow of energy onto or away from the brane-universe. The presentation is as follows: in section II we discuss the evolution of the background. We derive a Friedmann equation on the brane, which includes a term describing the energy flow from the fifth dimension. In section III we discuss the evolution of perturbations, and in particular we derive the equation of motion for the gauge invariant curvature variable  $\zeta$ . As we will show, this variable is not conserved even for adiabatic perturbations, in contrast to the “usual” four-dimensional world, where  $\zeta$  is conserved.

## II. BACKGROUND EVOLUTION OF THE BRANE UNIVERSE

In this paper we will consider only one brane, which describes our universe, embedded in a five-dimensional space. In particular, we are going to follow the formalism and notations introduced in [8]. The field equation is (we set the five-dimensional gravitational constant  $\kappa_5 \equiv 1$ ),

$$G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = T_{\alpha\beta} + T_{\alpha\beta}^{(b)}\delta(y - y_b). \quad (1)$$

where we consider a metric with the following form

$$ds^2 = a^2b^2 (dt^2 - dy^2) - a^2\Omega_{ij}dx^i dx^j, \quad (2)$$

with  $y$  denoting the coordinate along the fifth dimension, and  $\Omega_{ij}$  the metric of a 3-dimensional space of constant curvature, that is,

$$\Omega_{ij} = \delta_{ij} \left[ 1 + \frac{k}{4}x^l x^m \delta_{lm} \right]^{-2}, \quad (3)$$

where  $k = 0, 1, -1$  corresponds to a flat, closed or open universe, respectively. Also we assume that the two scale factors depend on both  $t$  and  $y$ , i.e.  $a = a(t, y)$  and  $b = b(t, y)$ . Note that we have chosen a conformal gauge for the  $t$ - $y$  part of the metric. In this gauge the brane is static and located at  $y = y_b$ , which we assume to be a *fix point* of a  $Z_2$ -symmetry along the fifth dimension (see [8] for details).

We shall make the mild assumption that the matter both on the bulk and brane is describable by a fluid. Thus, the energy-momentum tensor for the bulk matter is

$$T^\alpha{}_\beta = \begin{pmatrix} \rho_B & 0 & -r_B \\ 0 & -p_B\delta^i{}_j & 0 \\ r_B & 0 & -q_B \end{pmatrix}. \quad (4)$$

where  $r_B$  represents the energy flow onto the brane. This term is non-vanishing, for example, when there is a scalar field on the bulk which depends on both coordinates  $t$  and  $y$ . The brane matter is described by

$$T^{(b)}{}^\alpha{}_\beta = \begin{pmatrix} \rho & 0 & 0 \\ 0 & -p\delta^i{}_j & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (5)$$

For the sake of clarity, we shall denote bulk quantities with a subindex  $B$ , while quantities without subindex are brane quantities. Then, Einstein's equations for the metric (2) can be found to be

$$a^2b^2G^0{}_0 \equiv 3 \left[ 2\frac{\dot{a}^2}{a^2} + \frac{\dot{a}\dot{b}}{ab} - \frac{a''}{a} + \frac{a'b'}{ab} + kb^2 \right] = a^2b^2 [\rho_B + \rho\bar{\delta}(y - y_b)] \quad (6)$$

$$a^2b^2G^5{}_5 \equiv 3 \left[ \frac{\ddot{a}}{a} - \frac{\dot{a}\dot{b}}{ab} - 2\frac{a'^2}{a^2} - \frac{a'b'}{ab} + kb^2 \right] = -a^2b^2q_B \quad (7)$$

$$a^2 b^2 G^0_5 \equiv 3 \left[ -\frac{\dot{a}'}{a} + 2\frac{\dot{a}a'}{a^2} + \frac{\dot{a}b'}{ab} + \frac{a'\dot{b}}{ab} \right] = -a^2 b^2 r_B \quad (8)$$

$$a^2 b^2 G^i_j \equiv \left[ 3\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} - \frac{\dot{b}^2}{b^2} - 3\frac{a''}{a} - \frac{b''}{b} + \frac{b'^2}{b^2} + kb^2 \right] \delta^i_j = -a^2 b^2 [p_B + p\bar{\delta}(y - y_b)] \delta^i_j, \quad (9)$$

where the delta-function  $\bar{\delta}$  incorporates a factor  $1/ab$ , and we use the notation  $\dot{f} = \partial f / \partial t$ ,  $f' = \partial f / \partial y$ , for a given function  $f$ . Using standard methods, it is easy to derive the jump conditions for the metric components. We are going to assume here that our brane corresponds to the brane located at  $y = R$  in ref. [8]. These jump conditions are simply (see [8] for details)

$$\frac{a'}{a} = \frac{1}{6}ab\rho, \quad \frac{b'}{b} = -\frac{1}{2}ab(\rho + p). \quad (10)$$

Restricting the 05 component (8) to the brane and using the jump conditions above, we can easily find

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p) + 2abr_B. \quad (11)$$

This is the energy conservation equation on the brane, which has been generalized to allow a possible energy flow  $r_B$  onto the brane. The sign of  $r_B$ , according to (4), will determine if the energy flow is *onto* the brane or away from it. Equivalently, the restriction of the 55 component to the brane gives,

$$\frac{\ddot{a}}{a} - \frac{\dot{a}\dot{b}}{ab} + kb^2 = -\frac{a^2 b^2}{3} \left[ \frac{1}{12}\rho(\rho + 3p) + q_B \right]. \quad (12)$$

From these relations one can proceed to derive the Friedmann equation, *i.e.* to find a first integral of (12). We start by re-parametrising the time-coordinate on the brane as follows

$$d\tau = (ab)dt. \quad (13)$$

Thus, we change to the *proper time* on the brane (recall that on the brane the scale factors  $a$  and  $b$  are functions of  $\tau$  alone). Then, using the notation  $f_\tau = df/d\tau$  for a given function  $f$ , equation (12) changes to

$$\frac{a_{\tau\tau}}{a} + \left( \frac{a_\tau}{a} \right)^2 + \frac{k}{a^2} = -\frac{1}{3} \left[ \frac{1}{12}\rho(\rho + 3p) + q_B \right], \quad (14)$$

whereas the energy-conservation equation (11) now reads

$$\frac{d\rho}{d\tau} = -3\frac{a_\tau}{a}(\rho + p) + 2r_B. \quad (15)$$

Following [10], the easiest way to proceed is to write the scale factor  $a$  as  $a = \exp \alpha$ . Also, we use  $H = \dot{a}/a$  as the standard notation for the *Hubble parameter*, and for convenience we define another Hubble parameter referred to the proper time on the brane (13) as  $\mathcal{H} = a_\tau/a$ . Then, we easily get the following relations,

$$\frac{a_{\tau\tau}}{a} + \left( \frac{a_\tau}{a} \right)^2 = \alpha_{\tau\tau} + 2\dot{\alpha}_\tau^2, \quad (16)$$

and

$$2\alpha_{\tau\tau} = \frac{d\mathcal{H}^2}{d\alpha}, \quad (17)$$

so that one gets

$$\frac{d(\mathcal{H}^2 e^{4\alpha})}{d\alpha} = -e^{4\alpha} \left[ \frac{\rho(\rho + 3p)}{18} + \frac{2q_B}{3} \right] - \frac{2k}{e^{2\alpha}} e^{4\alpha}. \quad (18)$$

If equation (15) is now used to write the pressure  $p$  as a function of  $\rho$  and  $d\rho/d\tau$  and is then inserted in equation (18) one can easily derive

$$\frac{d(\mathcal{H}^2 e^{4\alpha})}{d\alpha} = \frac{d}{d\alpha} \left[ e^{4\alpha} \frac{\rho^2}{36} \right] - \frac{e^{4\alpha}}{9} \frac{r_B \rho}{\mathcal{H}} - \frac{2}{3} e^{4\alpha} q_B - \frac{d}{d\alpha} \left( e^{4\alpha} \frac{k}{e^{2\alpha}} \right). \quad (19)$$

Then, if we define two functions  $\mathcal{Q}$  and  $\mathcal{E}$  by

$$\frac{d}{d\alpha} e^{4\alpha} \mathcal{Q} = e^{4\alpha} q_B, \quad (20)$$

$$\frac{d}{d\alpha} e^{4\alpha} \mathcal{E} = e^{4\alpha} \frac{r_B \rho}{\mathcal{H}}, \quad (21)$$

we find the solution of eq. (19) to be

$$\mathcal{H}^2 = \frac{\rho^2}{36} - \frac{2}{3} \mathcal{Q} - \frac{1}{9} \mathcal{E} - \frac{k}{a^2} + \frac{\mathcal{A}}{a^4}, \quad (22)$$

where  $\mathcal{A}$  is an integration constant. Finally, it is useful to change to ordinary time derivatives in equations (20) and (21). This gives

$$\frac{d\mathcal{Q}}{d\tau} + 4\mathcal{H}\mathcal{Q} = \mathcal{H}q_B, \quad (23)$$

$$\frac{d\mathcal{E}}{d\tau} + 4\mathcal{H}\mathcal{E} = r_B \rho. \quad (24)$$

Equations (22)–(24) (together with (15)) are the evolution equations for the system we discuss. Furthermore, they allow a comparison to other existing approaches. Note that, for a scalar field in the bulk, the set of equations (22), (23) and (24) is equivalent to equation (12) in [11], written here in physical variables.

Inspired by heterotic M-theory or the Randall–Sundrum model, we may write  $\rho = \rho_M + U_B$ , where  $\rho_M$  is the “ordinary” brane matter (which by definition does not couple to the bulk matter) and  $U_B$  is a contribution from the (projected) bulk matter. It then follows that

$$\mathcal{H}^2 = \frac{\rho_M^2}{36} + \frac{1}{18} U_B \rho_M + \frac{U_B^2}{36} - \frac{2}{3} \mathcal{Q} - \frac{1}{9} \mathcal{E} - \frac{k}{a^2} + \frac{\mathcal{A}}{a^4}. \quad (25)$$

This equation has a similar form as in the Randall–Sundrum (RS) case. Indeed, to get the Friedmann equation, we just need to identify

$$48\pi G = U_B. \quad (26)$$

Furthermore, the split we have chosen ( $\rho = \rho_M + U_B$ ) suggests that both matter components evolve separately. In fact, by definition,  $U_B$  alone describes the interactions with the bulk. Hence the energy–conservation equation (15) splits into

$$\frac{dU_B}{d\tau} = -3\mathcal{H}U_B(1 + w_U(\tau)) + 2r_B, \quad (27)$$

$$\frac{d\rho_M}{d\tau} = -3\mathcal{H}(\rho_M + p_M). \quad (28)$$

Here,  $w_U$  may be seen as the effective equation of state of the (projected) bulk matter. Note that this provides a specific example of a model where a four-dimensional “effective” constant can evolve due to dynamical effects in higher dimensions. This fact alone may be a powerful tool in constraining brane–world models (see [12] and references therein).

For the particular case of  $r_B = 0$  and  $q_B = \text{const} = \rho_B$  (which represents the RS–case), we easily get from equation (25) and the definitions of  $\mathcal{Q}$  and  $\mathcal{E}$

$$\mathcal{H}^2 = \frac{\rho_M^2}{36} + \frac{8\pi G}{3} \rho_M + \Lambda_{\text{eff}} + \frac{\mathcal{C}}{a^4} - \frac{k}{a^2}, \quad (29)$$

which is in fact the equation found in the literature. Here, we have defined an effective cosmological term

$$\Lambda_{\text{eff}} = \frac{1}{6} \left( \frac{U_B^2}{6} - q_B \right). \quad (30)$$

Note that, in general, the integration constant  $\mathcal{C}$  is different from  $\mathcal{A}$ .

Some comments are in order at this point. In the model presented so far, the evolution of the induced gravitational constant  $G$  (26) is *universal*, if  $U_B$  (and thus  $\rho_B$ ) is not stabilized, i.e. if  $dU_B/d\tau \neq 0$ . However, we have two possibilities to stabilize  $G$ :

i) At late times, the equation of state of the projected bulk matter is  $w_U = -1$  and there is no energy flow, that is  $r_B = 0$ . This is in fact the case in Randall–Sundrum type models, in which, for example, the bulk is of anti-de Sitter type. Note that such a model allows for an early evolution of  $G$ , because it is conceivable that the anti-de Sitter space might be a natural endpoint of the evolution of the bulk.

ii) At late times (at least), the energy flow “tracks” the expansion of the brane universe, that is

$$r_B = \frac{3}{2} \mathcal{H} U_B (1 + w_U(\tau)) . \quad (31)$$

In such a model, it is imaginable that the bulk space reacts against changes of the dynamics on the brane, for example if the effective equation of state on the brane changes.

The first option has been discussed extensively in the literature. The second option, however, allows for several new insights in the stabilization of  $G$  in the framework of brane world theories. In particular, note that the dynamics of the five-dimensional bulk is contained in the quantity  $r_B$ , therefore the system of equations (22)-(24) and (27)-(28), which describe the dynamics on the branes, is *not* closed.

As a particular case, we are going to assume that we have a scalar field in the bulk. Such a situation arises, for example, in five-dimensional heterotic M-theory [13], or in dilatonic brane worlds [14] (see also [15]). We are going to denote the bulk-scalar field by  $\phi$ , its potential energy in the bulk by  $U$  and its projected energy on the brane by  $U_B$ . From the energy-momentum tensor of the bulk matter (see [13] for details) one finds

$$r_B = -T_5^0 = -\frac{\phi_\tau}{4} \frac{\partial U_B}{\partial \phi} , \quad (32)$$

and

$$q_B = -T_5^5 = -\frac{1}{4} \phi_\tau^2 + \frac{1}{16} \left( \frac{\partial U_B}{\partial \phi} \right)^2 - \frac{1}{2} U \quad (33)$$

In the expressions of  $r_B$  and  $q_B$  we have already used the boundary condition for the bulk scalar field on the brane. This condition, which can be found in ref. [13], is

$$\phi' = \frac{ab}{2} \frac{\partial U_B}{\partial \phi} . \quad (34)$$

From these expressions we can construct an effective cosmological constant in the Friedmann equation as follows. According to [14], and in order to define  $\Lambda_{\text{eff}}$ , we collect only those terms in the Friedmann equation which contain  $U$ ,  $U_B$  and the derivative  $\partial U_B / \partial \phi$ . Since the quantities  $U$  and  $U_B$  are slow time-varying for any physically motivated model, then we get from equation (23)

$$\Lambda_{\text{eff}} = \frac{1}{6} \left[ \frac{U_B^2}{6} + \frac{1}{2} U - \frac{1}{16} \left( \frac{\partial U_B}{\partial \phi} \right)^2 \right] . \quad (35)$$

The information that one can extract from this exercise is that the effective cosmological constant, as well as the induced gravitational constant  $G$  on the brane, are influenced by the 05- and the 55-component of the bulk energy-momentum tensor. In particular, the evolution of the induced gravitational constant is described by eqns. (26), (27) and (32). Note for instance that expression (35) for the cosmological constant is in agreement with [14].

Apart from the effective cosmological constant, the Friedmann equation on the brane contains an integral over  $\phi_\tau$ , coming from equation (23), and also a radiation term. Furthermore, the differential equation for  $\mathcal{E}$  (24) can be solved (see [14] for details). Thus, in [14] different solutions of dilatonic brane worlds were found. This is a path that we do not intend to pursue here.

To summarize, we have written the Friedmann equation in terms of  $q_B$  and  $r_B$ , which respectively are a time-integral of the 55-component of the bulk stress-energy tensor and a time-integral over the energy-flow onto the brane. Such a situation differs from the derivation in ref. [14] on the fact that in their case the bulk-dependence enters also through the projection of the five-dimensional Weyl-tensor on the brane.

### III. COSMOLOGICAL PERTURBATIONS AND THE BRANE/BULK INTERACTION

In the discussion so far we have restricted ourselves to the background evolution. However, as we will see in this section, interesting consequences appear also when considering cosmological perturbations. Our goal is to derive the equations of motion for the curvature perturbation in a rather general setup. The basic reason we have in mind to do this is the following. It was shown in [16] that the curvature of slices of constant density is conserved whenever energy conservation holds. Now, as discussed throughout the previous section, energy conservation in brane worlds is not warranted and an energy flow onto or away from the branes has important consequences. Given the results of [16], it is only natural to expect a modification of the evolution for super-horizon perturbations.

To discuss the evolution of density perturbations, we will choose a particular gauge to simplify the calculations. In [8] we have shown that in five dimensions we can use a “generalized” longitudinal gauge. We shall use the same gauge choice here. It defines a unique coordinate system and the advantages have been recognized in the literature. An important difference from the Randall–Sundrum gauge is that the perturbation of the 55-component is not zero. However, the perturbation variables coincide in this gauge with the gauge-invariant variables and thus we have control over gauge artifacts.

In what follows we will discuss scalar perturbations only. Moreover, we will limit ourselves to vanishing anisotropic stress (along the slices  $t = y = \text{constant}$ ) in the bulk as well as on the brane, which is the case for scalar fields. This assumption further ensures that the use of the “generalized longitudinal gauge” is appropriate.

With the assumptions listed above, the perturbed metric takes the form [8]

$$ds^2 = a^2 b^2 [(1 + 2\phi)dt^2 - (1 - 2\Gamma)dy^2 - 2Wdydt] - a^2 \Omega_{ij}(1 - 2\psi)dx^i dx^j. \quad (36)$$

The perturbed energy-momentum tensor has the form

$$\delta T^a{}_\beta = \begin{pmatrix} \delta\rho_B & -(\rho_B + p_B)b^{-2}v_{B|j} & -\delta r_B \\ (\rho_B + p_B)v_B^{|i} & -\delta p_B \delta^i{}_j & -u_B^{|i} \\ \delta r_B + 2r_B(\phi + \Gamma) - (\rho_B + q_B)W & -b^{-2}u_{B|j} & -\delta q_B \end{pmatrix} \quad (37)$$

for the bulk matter and

$$\delta T^a{}_\beta = \begin{pmatrix} \delta\rho & -(\rho + p)b^{-2}v_{|j} & -\delta r \\ (\rho + p)v^{|i} & -\delta p \delta^i{}_j & 0 \\ \delta r - \rho W & 0 & 0 \end{pmatrix} \quad (38)$$

for the brane matter.

The energy conservation for the brane-perturbations to first order can be found to be

$$(\delta\rho)^\cdot = -3H(\delta\rho + \delta p) + 3(\rho + p)\dot{\psi} - 2a^2(\rho + p)(\rho_B + q_B)v + 2ab\left(\Gamma + 2\phi + \frac{\delta r_B}{r_B}\right)r_B, \quad (39)$$

where we have neglected all spatial derivatives because we are interested in super-horizon amplitudes. All quantities are evaluated *on* the brane. This equation can be obtained by taking the jump of the 05-component of Einstein’s equation across the position of the brane (see [8] for details).

Eqn. (39) has the same form as in four dimensions, apart from the last two terms which describe the energy-flow onto or away from the brane. Of course, according to the equation above, the total energy (brane+bulk) is conserved. However, due to the inhomogeneities on the brane, the 5D geometry is distorted in such a way that gravitational energy from the extra dimension can flow onto the brane. In fact, such a statement is valid for any brane world scenario. Also, according to the setup we have used, equation (39) is general. Finally, recall that if the bulk is in its vacuum state (i.e.  $\rho_B + q_B = 0$  and  $r_B = 0$ ) and also  $\delta r_B = 0$ , then the usual 4D conservation law is valid.

Usually in the 4D case the matter perturbation is related to the metric perturbation via a gauge invariant variable  $\zeta$ , defined as [18]

$$\zeta = -\psi - H\frac{\delta\rho}{\dot{\rho}}. \quad (40)$$

This term makes sense in 5D cosmology under the assumptions that it is restricted on the brane and it transforms according to the induced 4D metric on the brane. Then, it is easy to calculate

$$\dot{\zeta} = -\dot{\psi} - H \frac{(\delta\rho)^\cdot}{\dot{\rho}} - 3H^2(1 + c_s^2) \frac{\delta\rho}{\dot{\rho}} - \frac{\delta\rho}{\dot{\rho}^2} [4abr_B\dot{H} - 2(abr_BH)^\cdot], \quad (41)$$

where we have used the energy conservation for the background (11) and the definition  $c_s^2 = \dot{p}/\dot{\rho}$ . Using the following definition for the pressure perturbation

$$\delta p = c_s^2 \delta\rho + \delta p_{\text{nad}}, \quad (42)$$

where  $\delta p_{\text{nad}}$  is the non-adiabatic part of the pressure perturbation on the brane (describing the entropy production), we get from eqns. (39) and (41)

$$\begin{aligned} \dot{\zeta} = & -\dot{\psi} \left[ 1 - \frac{1}{1-F} \right] - \frac{H}{1-F} \frac{\delta p_{\text{nad}}}{\rho + p} - \frac{1}{1-F} \left[ \frac{2}{3} a^2 v(\rho_B + q_B) \right] \\ & + \frac{2}{3(1-F)} \frac{ab}{\rho + p} \left( \Gamma + 2\phi + \frac{\delta r_B}{r_B} \right) r_B - \frac{\delta\rho}{\dot{\rho}^2} [4abr_B\dot{H} - 2(abr_BH)^\cdot], \end{aligned} \quad (43)$$

with  $F = 2abr_B/3H(\rho + p)$ .

There is important information coming from (43). Namely, as long as the extra dimension is not in a vacuum state (that is,  $\rho_B + q_B \neq 0$  and/or  $r_B \neq 0$ ), *the super-horizon amplitudes on the brane are not necessarily constant, even if the perturbations are adiabatic*. An amplification of perturbations was previously thought to happen at the end of inflation (preheating) [17]. In that case the second term in equation (43) would be important. However, in the case of brane worlds the amplification depends also on the history of the extra dimension. Thus, it is important to find out how and when the extra dimension becomes stabilized, as this can have obvious observational consequences.

In the case of the often discussed Randall–Sundrum world, the last three terms in (43) vanish (see also the discussion in [19]). More generally speaking, the bulk space has to settle into a vacuum state in the very early universe, otherwise the amplitude of adiabatic scalar perturbations will not remain constant. Models without this property will have extreme difficulties in surviving observational tests. However, for the particular case where the cosmological constant in the bulk is provided by a bulk scalar field (i.e.  $r_B = 0$  and  $\rho_B + q_B = 0$ ), the perturbation of the energy flow onto (or away from) the brane will be present (i.e. the  $\delta r_B$ -term). In that case, the evolution of a adiabatic perturbations on super-horizon scales will be described by

$$\dot{\zeta} = \frac{2}{3} \frac{ab\delta r_B}{\rho + p}. \quad (44)$$

This equation tells us that the perturbation of the energy flow will modify super-horizon perturbations on large scales, and provides a clear observational handle on these models.

Whereas our argumentation is based on the form of the energy–conservation on the brane it was shown recently by Gordon and Maartens that other bulk effects produce non-adiabatic modes on the brane, which implies an effective growth of super-horizon perturbations [20]. All these results make inflationary model building, among other things, considerably more restrictive than in the usual four–dimensional models.

#### IV. DISCUSSION

In this paper we have investigated the effects of the interaction between a brane universe and the bulk in which it is embedded. We have argued that in the case where the bulk scalar field contributes to the dynamics on the brane, the induced four–dimensional gravitational constant depends on the energy flux onto (or away) from the brane as well as on changes of the equation of state in the bulk.

Whenever a scalar field is in the bulk, an energy flow onto (or way from) the brane universe is possible. In these models, the dynamics of the scalar field has to be studied in detail, in order to make statements about the gravitational constant in four dimensions. Apart from the gravitational constant, an energy flux onto the brane changes its expansion dynamics. For example, one can even imagine models in which a cosmological constant and a black hole exist in the bulk [21]. In that case, the time–dependent mass of the black hole influences the dynamics on the brane.

The conclusions we have drawn hold strictly only for a single brane universe. There will be some changes if one considers more than one brane. In the case of two branes, for example, where energy is globally conserved and propagates between the two branes, the four–dimensional gravitational constant will depend on the boundary conditions *on both branes*. To what extent a bulk scalar field influences the dynamic on “our” brane world will thus

depend on the boundary condition on the other brane. But we expect that some of the statements remain valid in these models in the era where the universe is effectively five dimensional. Furthermore, we have assumed that the five-dimensional gravitational constant is time-independent.

We have also investigated the consequences of energy conservation violation for large scale perturbations in brane world models. We have shown that, in general, adiabatic super-horizon perturbations are not constant. The amplitude will depend crucially on the orbifold history and on the stabilization mechanism. We believe this will provide a powerful way to test and rule out many realizations of the ‘brane-world’ paradigm.

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